Using Tracking Differentiators in Designing Nonlinear Disturbance Observers for Uncertain Systems

Naser Kazemzadeh , Saeed Barghandan
Department of Electrical Engineering, Ahar Branch, Islamic Azad University, Ahar, Iran,
Email:nasser.kazemzadeh@yahoo.com , saeed_barghandan@yahoo.com (Corresponding author)

Abstract

In the present paper, a practical designing method has been proposed for a novel class of NDOs based on TD. Such NDOs can nearly estimate all uncertain disturbances (specifically disturbances without prediction information). Regarding the outstanding performance of TD filter not requiring a precise dynamic model, the proposed NDO can estimate external disturbance and not modeled dynamics. The results showed the superiority of the proposed method in estimating disturbance and improvement of control quality.

Keywords: non-linear systems, uncertain disturbances, non-linear disturbance observer (NDO), tracking differentiator

1- Introduction

It is clearly known that disturbance, uncertainty, and non-modeled dynamic exist in nearly all control systems. The controller should optimally resist against uncertainty. Therefore, in some recent years the resistant control of dynamic systems with uncertainties has been noticed.

The common method is to use resistant control [1-2], sliding mode control [3], and adaptive control [4]. These methods can be used in different types of uncertain systems. But they do have some faults such as: the reduction of control quality due to conservatism resulted from resistant control, chattering high input control frequency in sliding mode control technique, and parameter movement in adaptive control. It should be noted that if disturbances are estimated effectively and are compensated in control rules, the problems related to control systems with uncertainties may be alleviated more easily. Recently, non-linear observer status [5-6] has been widely used to revitalize the non-measured states of dynamic systems. If such an observer is used to estimate model uncertainties, it would be related to a class of non-linear disturbance observers (NDO). Many designing methods for NDOs have been posed to present an appropriate response to estimate disturbances for dynamic systems with uncertainties.

An NDO is designed based on structure system theory for dynamic systems with uncertainties with the least phase and the desirable relativity degree [7]. However, high and low disturbance limits should be predetermined and this limits the utility perspective of it seriously. In order to overcome such a weak point, another NDO has been proposed [9] and it has been successfully utilized in magnetic short saber system. This NDO does not need the previous information on high disturbance limits and their derivations. In [8], an NDO has been designed to control the robot. This NDO has been designed based on a non-linear presupposition through which the disturbance derivation is nearly equal to zero. Although the simulation and testing this NDO in [8] has been able to track several disturbances with time variables, it is impractical to presuppose that there would be a slower dynamic disturbance than dynamic NDO. On the contrary to the good results in disturbance estimation still there are some faults remarked in parts [7][9]. One of the worst cases is related to the over-estimation of disturbances considering the previous disturbance information that is usually calculated with great difficulty in practice. Another problem is that such NDOs are designed based on a precise dynamic model. Still, the measurement noise amount may have a negative effect on the precision of disturbance estimation. Therefore, the two new NDOs that require a precise dynamic model in [1][10], have been proposed for ultrasonic vehicles and robotic controls, respectively. The NDO designed in [10] is the development of the results in research [8]. The new NDO in [1] is also designed based on high rank sliding derivation proposed in [12-14]. Although without considering the measurement noise we can achieve a high estimation precision with this NDO [1], the designing parameters of NDO are achieved with lots of difficulty. Also, ith derivation of Lipsheets' that is achieved with great difficulties should be predetermined. Additionally, there is a common weak point between NDOs of [1][8]. If system status entails measurement noise, and most of the time it is the case and even it cannot be eliminated in engineering, estimation performance will become very weak.

By considering the works introduced, a class of NDOs was utilized in the present study based on a tracking derivator (TD). In first stage, a general designing method was designed for NDO based on TD and NDO stability was guaranteed by using TD and presupposing that the dynamic NDO is much faster than system dynamics. In second stage, another formulation of TD and NDO was explained. Finally, a controller based on dynamic inversion control (DIC) and sliding mode controller were proposed to control a sample system and the results of simulation were proposed to represent the efficiency of controllers and the proposed NDO.

2- Tracking Derivator

Extracting the derivations of non-consistent signals and signals encountering noise is an old and known problem [12] that has been noticed in recent years. high order sliding mode derivator is difficult to apply due to the need for high band Lipsheets' ith disturbance derivation. Another weakness of SDM is that its design parameters are adjusted with great difficulties. TD tracking derivator, as a new class of derivators, has first been proposed in [15]. This derivator can nearly alleviate all the problems with SMD and its filter performance is satisfactory. Recently, many TDs have been developed

TD is approved by using the theorem 1 [15] as follows:

Theorem 1: for the systems expressed as follows:

$$\begin{cases} \dot{z}_1 = z_2 \\ \dot{z}_2 = F(z_1, z_2) \end{cases} \quad (1)$$

Where, $z_1 \in R$, $z_2 \in R$, presuppose that the responses of the system (1) with conditions $z_1(t) \to 0$, $z_2(t) \to 0$, we have $t \to \infty$ guranteed. Therefore, for each input function desired where v is limited and could be integralized and we have $T > 0, R > 0$, the response of the following system:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = R^2 \, F\left(x_1 - v(t), \frac{x_2}{R}\right) \end{cases} \quad (2)$$

Would gurantee the following condition:

$$\lim_{R\to\infty} \int_0^T |x_1 - v(t)| dt = 0 \quad (3)$$

Theorem 1 shows that $x_1(t) \in R$ is usually congruent with the input signal of $v(t)$. $x_2(t) \in R$ is weakly congruent with the derivation of $v(t)$. The performance of TD depends on F function and R parameter. Also the amount of R affects the congruence speed of TD.

Theorem 1 presents an overal TD designing method that makes system (1) states in base coordinates of $(0,0)$ asymptote by choosing a certain type of F function. Therefore, a type of TD with different efficiencies such as rapid congruence, without chatering, and noise limitation have been developed [18] (see (4)).

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = R^2 \left(-a_0(x_1 - v) - a_1(x_1 - v)^{\frac{m}{n}} - b_0\left(\frac{x_2}{R}\right) - b_1\left(\frac{x_2}{R}\right)^{\frac{m}{n}}\right) \end{cases} \quad (4)$$

Where, $a_0 > 0$, $R > 0$, $a_1 > 0$, $b_0 > 0$, $b_1 > 0$ are designing parameters and $0 < n < m$ are odd numbers. The consistency and congruence of the system [4] were approved in details in [15]. On the contrary to the presence of advantages such as rapid overal congruence and great capability in filtering the noise, this NDO is practicaly difficult to apply due to having many designing parameters.

3- Designing Non-linear Disturbance Observer Based on Tracking Derivator

Without losing the totality of the system, the uncertain dynamic system proposed below is taken into consideration:

$$\dot{x} = f(x) + g(x)u + d \quad (5)$$

Where, $x \in R$, $u \in R$ and they are the state and input in control system, respectively. Also $d \in R$ represents the disturbance.

In order to alleviate the faults in introduced NDOs in [7-11], a new class of NDO has been introduced in this research as follows:

Theorem 2: For the system below:

$$\begin{cases} \dot{\hat{x}} = f(x) + g(x)u + \hat{d} \\ \dot{\hat{d}} = R^2 F(\hat{x} - x, \hat{d}/R) \end{cases} \quad (6)$$

Where, $\hat{x}$ and $\hat{d}$ are the estimation of x and d.

If $R > 0$, and $T > 0$, we would have:

$$\lim_{R\to\infty}\int_0^T |\hat{x}-x|dt = 0 \qquad (7)$$

Where, $\hat{x} \to x$. Additionally, regarding the equations (5) and (6), surely we would have:

$\hat{d} \to d$.

Proof

When $R \to \infty$ and $\left|\dot{\hat{d}}\right| = \left|R^2F\left(\hat{x}-x,\frac{\hat{d}}{R}\right)\right|$ may become bigger unlimitedly. Therefore, $\hat{d}$ would change much more rapidly than $f + gu$. Also we have $\lim_{R\to\infty}\frac{d(f+gu+\hat{d})}{dt} = \dot{\hat{d}}$ and $\lim_{R\to\infty}\frac{(f+gu+\hat{d})}{R} = \frac{\hat{d}}{R}$. Thus, the equations (6)-(7) will be correct by presupposing $x_2 = f + gu + \hat{d}$.

Remark 1: It should be noted that in a real system, control input is very limited. In this way, bandwidth and the speed of changes in u is limited. Therefore, the presupposition that states $\hat{d}$ to change much more rapidly than $f + gu$ is reasonable.

The special innovation in theorem 2 refers to the fact that an overall NDO designing method is represented based on TD. Thus, we can develop different NDOs by appropriate TD selection. The outstanding advantage of NDO (6) is that we can nearly estimate all different types of disturbances without any forecast information. If the calculated TD in (6) has low noise capability, there will be a satisfactory performance for NDO.

Let's presuppose that in system (5), uncertainty exists in a system as follows:

$$\begin{aligned}\dot{x} &= [f(x)+\Delta f(x)] \\ &\quad +[g(x)+\Delta g(x)]u + d_0 + d\end{aligned} \qquad (8)$$

Where, $\Delta f\,(x)$ and $\Delta g\,(x)$ represent disturbance and interferences related to $f(x)$ and $g(x)$ and d_0 represents external disturbance and the un-modeled dynamic in system (5), $g\,(x) \neq 0$.

Therefore, system (8) can be written as follows:

$$\dot{x} = f(x) + g(x)u + d_{all} \qquad (9)$$

Where, $d_{all} = \Delta f(x) + \Delta g(x)u + d_0 + d$ shows total disturbances. Therefore, NDO (5) can be revised as follows:

$$\begin{cases}\dot{\hat{x}} = f(x) + g(x)u + \hat{d}_{all} \\ \dot{\hat{d}}_{all} = R^2F\left(\hat{x}-x, \hat{d}_{all}/R\right)\end{cases} \qquad (10)$$

Equations (8)-(10) show that the proposed NDO can also estimate the un-modeled dynamics and external disturbances. Thus, it will be robust regarding the un-modeled disturbances. The NDO equations based on the TD proposed in section 3 and Theorem 2 were developed as equation (11).

Where, $a_0 > 0,\ R > 0,\ a_1 > 0,\ b_0 > 0,$ and $b_1 > 0$ are designing parameters and $0 < n < m$ are odd numbers. The practical adaptation rules of the designing parameters for the proposed NDO can be found in [16, 18] and [23].

$$\begin{cases}\dot{\hat{x}} = f(x) + g(x)u + \hat{d} \\ \dot{\hat{d}} = R^2\left(-a_0(\hat{x}-x) - a_1(\hat{x}-x)^{\frac{m}{n}} - b_0\left(\frac{\hat{d}}{R}\right) - b_1\left(\frac{\hat{d}}{R}\right)^{\frac{m}{n}}\right)\end{cases} \qquad (11)$$

4- Inversed Dynamic Controller

The identified control goal in this section is the direction of x state to track an x_d by adjusting input control u. the inversed dynamic control for the system has been designed with an unidentified dynamic (5).

To do so, the tracking error is defined as follows:

$$e = x - x_d \tag{12}$$

Derivations from (10) regarding equation (4) leads to:

$$\dot{e} = f(x) + g(x)u + d - \dot{x}_d \tag{13}$$

Control input u can be selected as follows:

$$u = g^{-1}(x)\left(-\Gamma_1 e - \Gamma_2 \int_0^t e\, d\tau - f(x) - \hat{d} + \dot{x}_d\right) \tag{14}$$

Where, $\hat{d}$ represents d estimation by NDO and $\Gamma_2 > 0$, $\Gamma_1 > 0$ are designing parameters. A candidate Liapanoph function can be selected as follows:

$$W = \frac{e^2}{2} + \frac{\Gamma_2}{2}\left(\int_0^t e\, d\tau\right)^2 + \frac{\tilde{d}^2}{2} \tag{15}$$

Where, $\tilde{d} = \hat{d} - d$. By calculating the derivation from the equation (13) we will have:

$$\begin{aligned} \dot{W} &= -\Gamma_1 e^2 - e\,\tilde{d}_i + \tilde{d}\dot{\tilde{d}} \\ &< -\Gamma_1 e^2 + \frac{(e\,\tilde{d}_i)^2}{2} + \frac{1}{2} + \tilde{d}\dot{\tilde{d}} \\ &= -(\Gamma_1 - \frac{\tilde{d}_i^2}{2})e^2 + \frac{1}{2} + \tilde{d}\dot{\tilde{d}} \end{aligned} \tag{16}$$

Since regarding Theorem 2, $\left|\frac{1}{2} + \tilde{d}\dot{\tilde{d}}\right|$ is limited. By the selection above, $\left|\frac{1}{2} + \tilde{d}\dot{\tilde{d}}\right| < M$, if $\Gamma_1 > \frac{M}{e^2} + \frac{\tilde{d}_i^2}{2}$, there will be $W < 0$. Therefore, the error dynamic is consistent.

5- Sliding Mode Controller

In this part a sliding mode controller has been designed for indefinite dynamic system (5) to investigate about the effect of the proposed NDO in reducing chattering. We know that usually there is an unwanted sliding mode controller with a high frequency. This chattering can be reduced considerably by using the proposed NDO.

To design the sliding mode controller we consider a sliding level as follows:

$$s = x - x_d \tag{17}$$

By calculating the derivation of (17) and regarding (5) we will have:

$$\dot{s} = f + gu + d - \dot{x}_d \tag{18}$$

The control input maybe selected as follows:

$$u = g^{-1}(-\Gamma s - f - \rho\, sign(s) - \hat{d} + \dot{x}_d) \tag{19}$$

In a way that $\hat{d}$ represents d by NDO and $\Gamma > 0$, $\rho > 0$ are designing parameters. A candidate Liapanoph function can be selected as follows:

$$W = \frac{1}{2}s^2 + \frac{1}{2}\tilde{d}^2 \tag{20}$$

In a way that $\tilde{d} = \hat{d} - d$. By calculating the derivation of equation (18) we will have:

$$\dot{W} = -\Gamma s^2 - \rho\,|s| - s\tilde{d}_i + \tilde{d}\dot{\tilde{d}} \tag{21}$$

If $\rho > |\tilde{d}|$, by the selection of high bandwidth $\left|\tilde{d}\dot{\tilde{d}}\right|$ equal to μ, equation (19) will be as follows:

$$\dot{W} < -\Gamma s^2 + \mu \tag{22}$$

If $\Gamma > \mu/s^2$ is selected, there would be $\dot{W} < 0$ and tracking error dynamic of s would be consistent.

6- Simulation (Sliding Mode Controller)

In this part a sliding mode controller has been designed for uncertain dynamic system (5) to investigate about the effect of the proposed NDOs in reducing chattering. We know that usually there is an unwanted sliding mode controller with a high frequency. This chattering can be reduced considerably by using the proposed NDO. Next, a simulation was supplied to show the efficiency of the proposed NDO to reduce the input chattering.

In this simulation we presuppose that $x_d = sin(t)$, $f = x^2 + sin(x) + 1$, $g = 5$, $\Gamma = 10$, $d = 0.1\, sin(t)$, and $\rho = 1$. NDO designing parameters were: $b_0 = 1$, $R = 10$, $a_0 = a_1 = 5$, $b_1 = 1.14$, $m = 9$, $n = 3$. Now we repeat the simulation to change different parameters of simulation NDO to investigate about their effects.

6-1- Studding the effects of parameter R

In this part, the system performance will be studied for the changes of R parameter. (Other NDO designing parameters were selected in the form of $a_0 = a_1 = 5$, $b_0 =1$, $b_1 = 1.14$, $m = 9$, $n = 3$, and $\rho = 1$).

The results of simulations in figures 1 and 2 show the high estimation and tracking precision calculated by NDO. Also estimation and tracking performances have become better by increasing R.

Results of simulations are represented in figures 3 and 4. Tracking performance becomes better by increasing ρ, but as was is expected it did not have much effect on disturbance estimation.

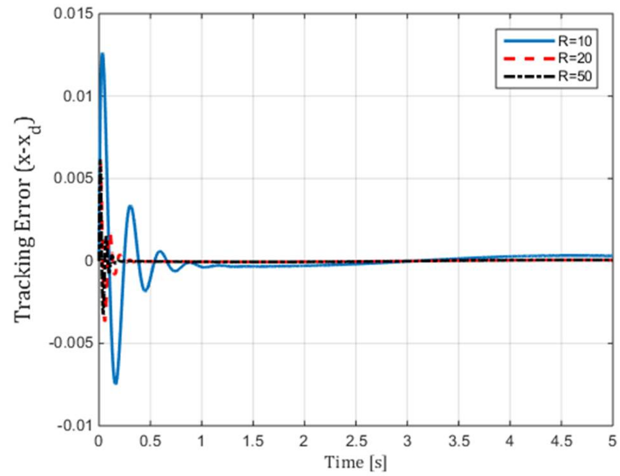

Fig.1. State tracking error for different amounts of R

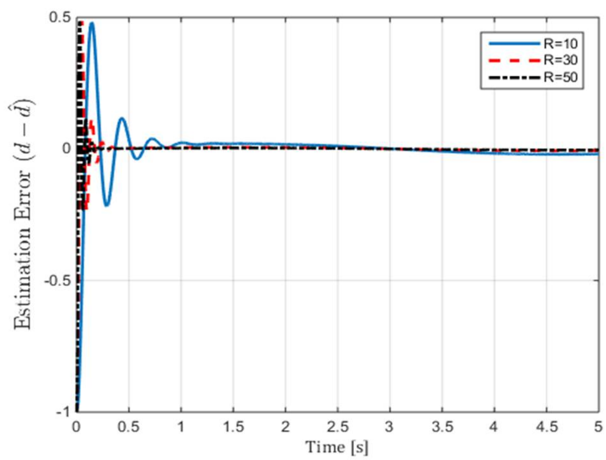

Fig.2. Disturbance estimation error for different amounts of R

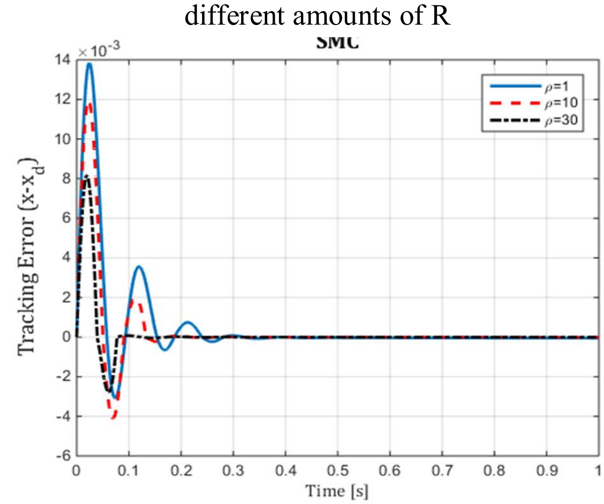

Fig.3. State tracking error for different amounts of R

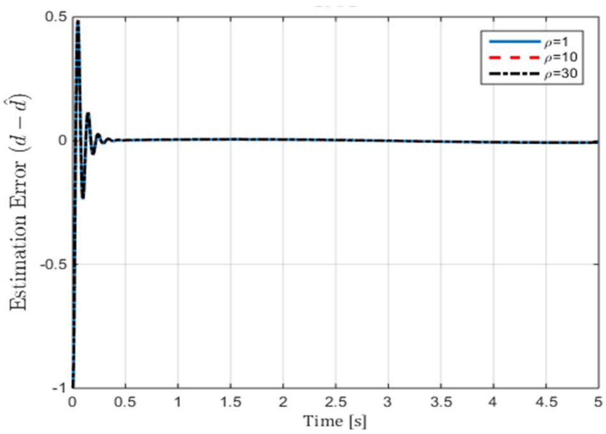

Fig.4. Disturbance estimation error for different amounts of R

7- Simulation (Reversed Dynamic Controller)

The goal of controller considered in this section is to direct state x to track x_d of the referenced signal by adjusting control input u. The reversed dynamic control for the system has been designed by using an uncertain dynamic (4). Then, simulation to validate the proposed NDO was presented. We presupposed that: $x_d = sin(t)$, $f = x^2 + sin(x) + 1$, $g = 5$, $\Gamma_1 = 10$, $\Gamma_2 = 1$, and $d = sin(t)$. Also other NDO designing parameters were selected as follows: $a_0 = a_1 = 5$, $b_0 = 1$, $b_1 = 1.14$, $m = 9$, $n = 3$.

7-1- Studding the effects of parameter R

We investigated the system performance for changes in R parameter.

7-2- Studding the effects of parameter Γ_1

Below the performance of closed loop system for the changes of parameter Γ_2 has been investigated ($R = 50$).

By increasing the amount of Γ_1 parameter, the system performance becomes better but the difference in system behavior does not have sensible changes for the amounts greater than 10.

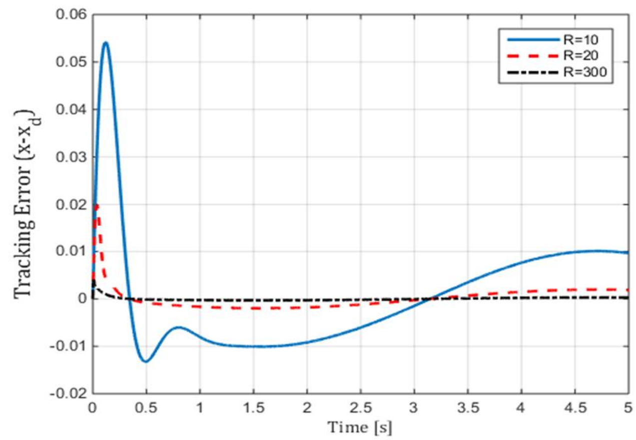

Fig.5. State tracking error for different amounts of R

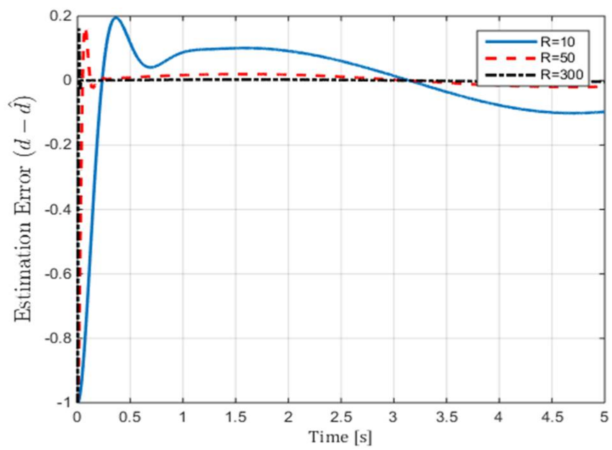

Fig.6. Disturbance estimation error for different amounts of R

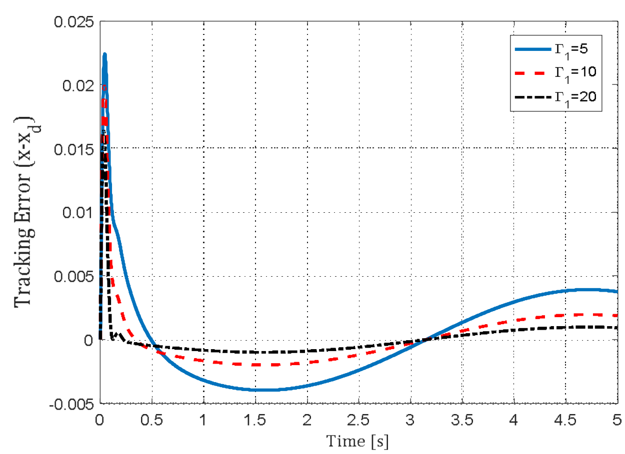

Fig.7. State tracking error for different amounts of Γ_1

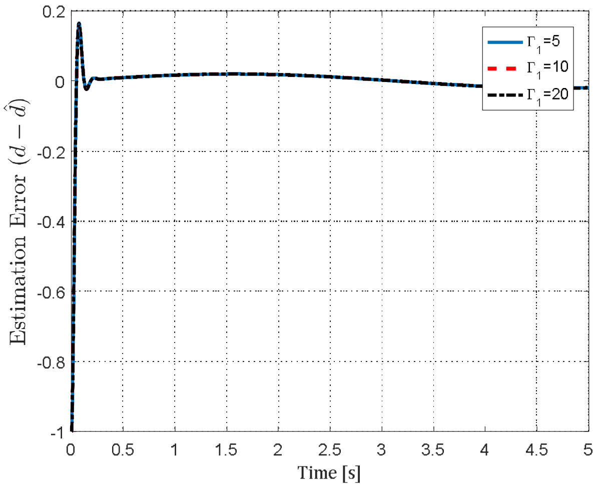

Fig.8. Disturbance estimation error for different amounts of Γ_1

7-3- Studding the effects of parameter Γ_2

Below the performance of closed loop system for the changes of parameter Γ_2 has been investigated ($R = 50$).

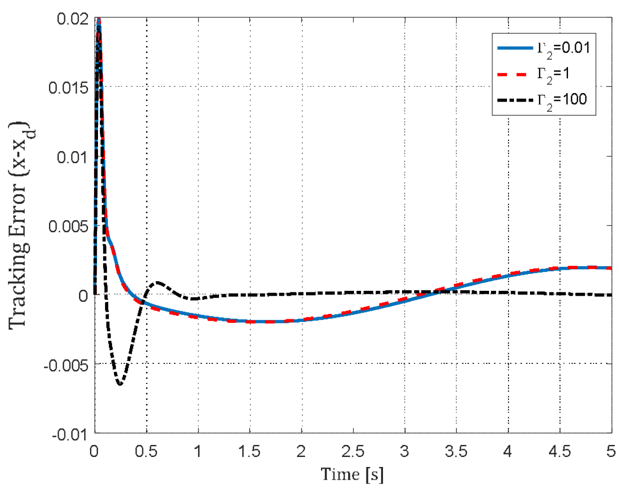

Fig.9. State tracking error for different amounts of Γ_2

The system performance for $\Gamma_2 = 1$ has an optimal state in a way that by increasing the amount to higher than that the tracking will encounter a great movement.

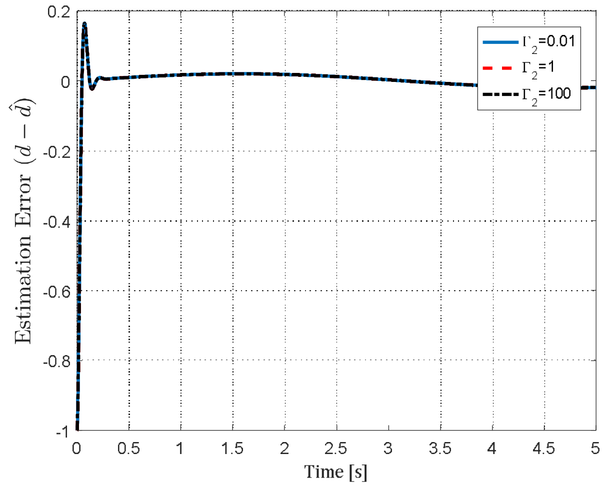

Fig.10. Disturbance estimation error for different amounts of Γ_1

8- Conclusion

In the present study, a practical designing method was proposed for a new class of NDOs based on TD. Such NDOs can almost estimate all types of uncertain disturbances (specifically disturbances without previous data). Regarding the optimal performance of TD filter without requiring a precise dynamic model, the proposed NDO can estimate external disturbances and un-modeled dynamics. Results of simulation showed the efficiency of the proposed method in disturbance estimation and the enhancement of control quality.

References

[1] J. Yang, S. H. Li, C. Y. Sun, and L. Guo, "Nonlinear-disturbance-observer-based robust flight control for air breathing hypersonic vehicles," IEEE Trans. on Aerospace and Electronic Systems, April 2013, vol. 49, no. 2, pp. 1263-1275.

[2] G. Gao and J. Z. Wang, "Reference command Tracking control for an air-breathing hypersonic vehicle with parametric uncertainties", Journal of the Franklin Institute, 2013,vol. 350 , pp. 1155-1188.

[3] M. C. Pai, "Observer-based adaptive sliding mode control for robust tracking and model following," International Journal of Control, Automation, and Systems, April 2013,vol. 11, no. 2, pp. 225-232.
[4] X. W. Bu, X. Y. Wu, Y. X. Chen, and R. Y. Bai, "Adaptive backstepping control of hypersonic vehicle based on nonlinear disturbance observer," Journal of National University of Defense Technology, October 2014,vo1. 36, no. 5, pp. 44-49.
[5] G. W. Fan, Z. Y. Liu, and H. Chen, "Controller and observer design for a class of discrete-time nonlinear switching systems," International Journal of Control, Automation, and Systems, December 2012,vol. 10, no. 6,pp. 1193-1203
[6] B. Y. Huo, S. C. Tong, and Y. M. Li, "Observer based adaptive fuzzy fault-tolerant output feedback control of uncertain nonlinear systems with actuator faults," International Journal of Control, Automation, and Systems, December 2012 .
[7] X. K. Chen, S. Komada, and T. Fukuda, "Design of a nonlinear disturbance observer," IEEE Trans. on Industrial Electronics, April 2000,vol. 47, no. 2, pp. 429-437.
[8] W. H. Chen, D. J. Ballance, P. J. Gawthrop, and J.O' Reilly, "A nonlinear disturbance observer for robotic manipulators," IEEE Trans. on Industrial Electronics, August 2000,vol. 47,no. 4,pp. 932-938.
[9] X. K. Chen, C. Y. Su, and T. Fukuda, "A nonlinear disturbance observer for multivariable systems and its application to magnetic bearing systems," IEEE Trans. on Control Systems Technology, July 2004, vol. 12, no. 4, pp.569-577.
[10] A. Mohammadi, M. Tavakoli, H. J. Marquez, and F. Hashemzadeh, "Nonlinear disturbance observer design for robotic manipulators," Control Engineering Practice, 2013, vol. 21, pp. 253-267 **.**
[11] S. S. Yeh, M. F. Tu, and K. S. Tseng, "A simplified approach to the design of nonlinear disturbance observer for axial motion systems," Proc. of the IEEE International Conference on Mechatronics, 2005 ,pp. 10-12 .
[12] A. Levant, "High-order sliding models, differentiation and output-feedback control," International Journal of Control, 2003, vol. 76, pp. 924-941 .
[13] A. Levant, "Quasi-continuous high-order sliding mode controllers," IEEE Trans. on Automatic Control, November 2005,vol. 50, no. 11, pp. 1812-1816 .
[14] Y. B. Shtessel, I. A. Shkolnikov, and A. Levant, "Guidance and control of missile interceptor using second-order sliding modes," IEEE Trans. on Aerospace and Electronic Systems, January ,2009, vol. 45, no. 1, pp.110-12
[15] J. Q. Han and W. Wang, "Nonlinear tracking differentiator," Journal of Systems Science and Mathematical Sciences, April1994 , vol. 14, no. 2, pp. 177-183.
[16] G. Y. Qi, Z. Q. Chen, and Z. Z. Yuan, "New tracking-differentiator design and analysis of its stability and convergence," Journal of Systems Engineering and Electronics, August 2004 , vol. 15, no. 4, pp. 780-787
[17] H. Feng and S. J. Li, "A tracking differentiator based on Taylor expansion," Applied Mathematics Letters, 2013, vol. 26, pp. 735-740.
[18] Y. L. Shi and C. Z. Hou, "Design of improved nonlinear tracking differentiator," Control & Decision, June 2008,vol. 23, no. 6, pp. 647-650.
[19] X. H. Wang, Z. Q. Chen, and G. Yang, "Finite time-convergent differentiator based on singular perturbation technique," IEEE Trans. on Automatic Control, September2007, vol. 52, no. 9, pp. 1731-1737.
[20] Y. G. Tang, Y. X. Wu, M. P. Wu, X. P. Hu, and L.C. Shen, "Nonlinear tracking-differentiator for Velocity determination using carrier phase measurements," IEEE Journal of Selected Topics in Signal Processing, August 2009, vol.3, no. 4, pp. 716-725.
[21] X. H. Wang and B. Shirinzadeh, "Rapid-convergent nonlinear differentiator," Mechanical Systems and Signal Processing, 2012, vol. 28, pp. 414-431.
[22] D.V. Efimov and L. Fridman, "Ahybrid robust nonhomogeneous finite-time differentiator," IEEE Trans. on Automatic Control, May 2011, vol.56, no.5, pp.1213-1219.
[23] X. M. Dong and P. Zhang, "Design and phase plane analysis of an arctangent based tracking differentiator," Control Theory & Applications, April 2010, vol. 27, no. 4, pp. 533-537.