

# Sliding Mode Control for Single-degree-of-freedom Nonlinear Structures

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## Abstract

*The occurrence of catastrophic earthquakes necessitates further researches in the structure engineering for retrofitting construction structures. In this paper, the application of the active control in the structures' seismic response has been addressed. A single degree of freedom nonlinear structure has been studied. The nonlinear dynamic of the structure is considered in which, the nonlinear part of the dynamic is modeled by Bouc-Wen model. The sliding mode controller is used to stabilize the system. The results show the effectiveness of the proposed method.*

**Keywords:** active control of structures, sliding mode control, Bouc-Wen equation

## 1. Introduction

In fact, structure controlling refers to control and reduce structure responses under the pressure of loads (specifically peripheral loads such as earthquake load and wind load) enforced throughout the structure lifespan. Structure controlling is a novel branch of science in structure engineering and it has been noticed by the researchers during some recent decades.

In fact, considering a structure as a dynamic system, some of the characteristics attributed to it such as stiffness and attenuation of it can be adjusted in a way that the dynamic effect of the forces enforced on the structure can be reduced to an acceptable level. The issue of structure controlling is divided into three methods including active control, inactive control, and semi-active control.

The idea of a structure's active control was systematically proposed for the first time in year 1972 by Yao and it was considered as a

starting point for research carried out on active control of the structures [1].

In an active control system always we need a great source of energy to install electro-mechanic stimulants or electro-hydraulic systems which enforce controlling forces onto the structure. The controlling forces are created based on the feedback resulted from the sensors through measuring structure responses or through the stimulation enforced. Since the active control systems require an external energy resource to perform better, it would be necessary to keep this energy resource unchanged and unhurt throughout the severe incidents happening to avoid lack of structure unity and lowering the performance of it. Furthermore, there is a probability that by applying surplus mechanical forces onto the structure in active control systems, the structure will encounter inconsistency. Therefore, active control systems are principally used as complimentary systems for inactive control systems in engineering structures.

We can refer to the role of active mass mirages in reducing building vibrations through strong winds and medium earthquakes as an example of the functions of such controls [2].

Marti considered active control strategies in order to achieve consistency in mechanical structures under the pressure of dynamic loads.

The structures are usually fixed, safe and consistent, without external dynamic disturbances such as strong earthquakes, tornadoes, and twirling waters. The goal of an active control for mechanical structures is to evade mechanical forces applied such as earthquakes, winds, and water waves that lead to probable losses in structures. Mechanical structure modeling has been represented through a first degree random differential system for the status vector ( $q$  movement vector and  $\dot{q}$  time derivation) and the pre-determined optimized control in order to encounter with random non-absoluteness through forces applied [3].

In the present research we have used non-linear analysis of a vibrating mode controller considering Bouc-Wen model for designing and simulating a structure with first freedom degree. Also we have used El-Centro and Erzincan earthquakes as two examples of important recorded earthquakes to represent the performance of the proposed method.

## 2. Experimental literature

The mechanical behavior of the structures can be modeled by using the linear integration of masses, stiffness, and attenuation.

In this case, if we represent the applied force onto the structure as  $u(t)$ , the application of such a force will lead the structure to move horizontally and we can represent structure relocation in any time compared to the earth level using  $x(t)$ .

$$M\ddot{x} + C\dot{x} + Kx = u(t) \quad (1)$$

In equation (1),  $x(t)$  shows location change,  $\dot{x}(t)$  shows speed,  $\ddot{x}(t)$  represents velocity, and  $u(t)$  means the external dynamic force vector.  $M$  is structure mass,  $C$  represents attenuation, and  $K$  shows structure strength [4].

In reality, when a structure is affected by severe earthquakes, it shows movements forwards and backwards and the force vector based on location change represents the formation of hysteresis loops [5]. Therefore, the dynamic behavior of a structure is a nonlinear behavior through which there are hysteresis loops.

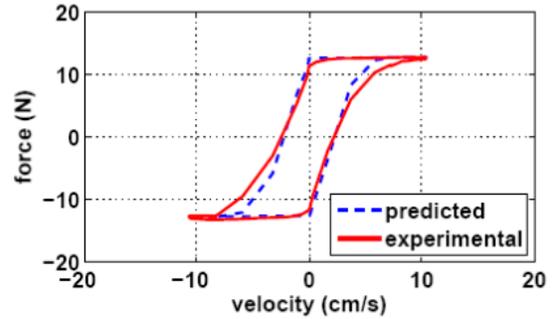


Fig. 1. Hysteresis loops of Bouc-Wen model [6]

Considering the model above, the dynamic equation dominating the one story structure are identified through certain parameters as follows:

$$m\ddot{x} + c\dot{x} + \varphi(x, \dot{x}) = u(t) + f(t) \quad (2)$$

Where,  $m$  and  $c$  represent mass and stiffness of the structure and  $f(t)$  and  $u(t)$  show chaos force and the control force

applied onto the structure, respectively. Represents the hysteresis behavior and it is modeled using Bouc-Wen modeling equation as follows:

$$\varphi(x, x) = \alpha kx(t) + (1 - \alpha) Dkz(t) \quad (3)$$

$$\dot{z} = D^{-1} \left( A\dot{x}(t) - \beta |\dot{x}| |z(t)|^{n-1} z(t) - \lambda \dot{x}(t) |z(t)|^n \right) \quad (4)$$

Where, the hysteresis behavior between the variables of  $x(t)$  and  $z(t)$  are taken into consideration. In equation 3,  $\varphi$  is written as the sum of an elastic element of  $\alpha kx(t)$  and an element of  $(1 - \alpha) Dkz(t)$  hysteresis. In this equation,  $D > 0$  is known as yield constant displacement and  $0 < \alpha < 1$  is post to pre-yielding stiffness ratio. In equation (4), the parameters of  $A$ ,  $\beta$ , and  $\lambda$  indexes without any dimension identify the hysteresis convex and  $n \geq 1$  refers to the dimension through which the hysteresis convex constantly controls the elastic to plastic states. Through the selection of different amounts of the parameters in Bouc-Wen equation we can model a vast spectrum of hysteresis loops [7]. We can benefit from the bank characteristics of the variable of hysteresis loops and consider it as a variable similar to the effect of earthquake in the form of a chaos with banks.

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$$m\ddot{x} + c\dot{x} + k\alpha_0 x = \underbrace{f(t) - (1 - \alpha_0) Dkz(t)}_{\tilde{f}(t)} + u(t) \quad (5)$$

In fact, the definition leads to:

$$\tilde{f}(t) = f(t) - (1 - \alpha) Dkz(t) \quad (6)$$

The dynamic behavior of the nonlinear system identified in equations (2) to (4) is changed into a linear system. The difference lies in the fact that in the new system, the disturbances applied onto the system are caused by the two elements of earthquake and hysteresis variables. The high bank of chaos in  $f(t)$  can be represented as follows:

$$\begin{aligned} \tilde{f}_{\max} &= \max_t \{ \tilde{f}(t) \} \\ &= \max_t \{ \{ (-ma(t)) \} \} + \max_t \{ \{ (1 - \alpha) Dkz(t) \} \} \\ &= ma_{\max} + (1 - \alpha) Dkz_{\max} \end{aligned} \quad (7)$$

Regarding the fact that hysteresis variable  $z(t)$  can have banks through applying one of the four conditions of the first four classes in table 1. For designing the controller we can use equation (3) in the form of the sum of  $\alpha kx(t)$  (which includes the variables of system states) and  $(1 - \alpha) Dkz(t)$  which includes a variable with banks.

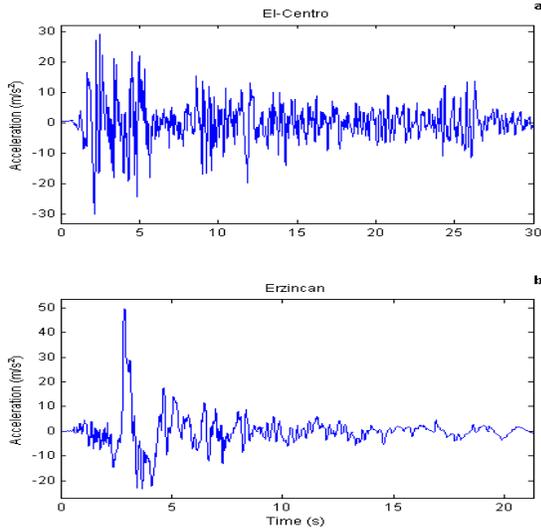
Then we can use the bank characteristic of  $(1 - \alpha) Dkz(t)$  to consider it similar to the effect of earthquake as an external disturbance

To simulate the intended controller we have considered mass, stiffness and attenuation of the structure equal to  $m = 360000(kg)$ ,  $(Ns / m)c = 25000$ ,  $k = 2000000(N / m)$ , respectively.

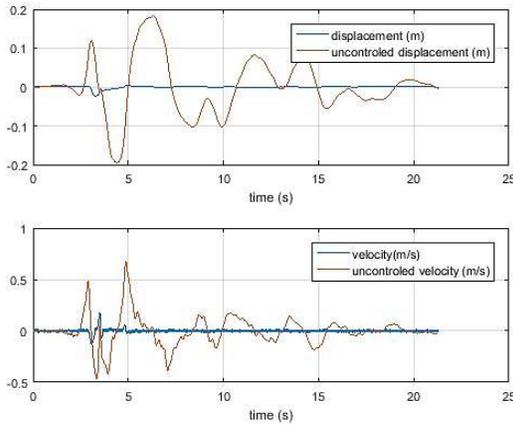
The parameters of the equation proposed by Bouc-Wen were selected as follows:  $A = 1$ ,  $\beta = 2.2$ ,  $n = 1$ ,  $\lambda = 0.5$ ,  $D = 0.6$ ,  $\alpha = 0.2$

### 3. Results and Discussion

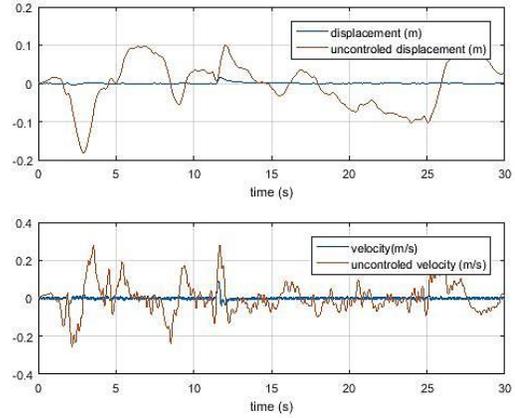
The graph of velocity of earthquakes considered as represented in figure 2.



**Fig. 2-a.** Velocity of El-Centro earthquake  
**2-b.** Velocity of Erzincan earthquake



**Fig. 3.** Displacement and speed change of the structure affected by Erzincan earthquake



**Fig. 4.** Displacement and speed change of the structure affected by El-Centro earthquake

**Table.1.** Bouc-Wen equation regarding the consistency of BIBO

		$\Omega$	$ z(t) $ upper bound	Class
$A > 0$	$\beta + \gamma > 0, \beta - \gamma \geq 0$	R	$\max( z(0) , z_0)$	I
	$\beta - \gamma < 0, \beta \geq 0$	$[-z_1, z_1]$	$\max( z(0) , z_0)$	II
$A < 0$	$\beta - \gamma > 0, \beta + \gamma \geq 0$	R	$\max( z(0) , z_1)$	III
	$\beta + \gamma < 0, \beta \geq 0$	$[-z_0, z_0]$	$\max( z(0) , z_1)$	IV
$A = 0$	$\beta + \gamma > 0, \beta - \gamma \geq 0$	R	$ z(0) $	V
others		$\phi$		

### Conclusion

In this research, first the dynamics of nonlinear one story structures were modeled. Then the consistency of BIBO regarding the equation of Bouc-Wen was investigated considering a certain set of parameters. Then the controller was designed. The results showed the effectiveness of the method proposed and the responses of displacement and speed of the controlled structure was compared to the uncontrolled structures and they showed a considerable reduction in speed.

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