

# Accelerating Magnetic Resonance Imaging through Compressed Sensing Theory in the Direction space-k

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## Abstract

*Magnetic Resonance Imaging (MRI) is a noninvasive imaging method widely used in medical diagnosis. Data in MRI are obtained line-by-line within the K-space, where there are usually a great number of such lines. For this reason, magnetic resonance imaging is slow. MRI can be accelerated through several methods such as parallel imaging and compressed sensing, where a fraction of the K-space lines is obtained. According to the advanced mathematical theories about compressed sensing, images entailing sparse representation within a certain area can be restored through a random subsampling of K-space data. MRI images are often sparse in an appropriate conversion range, where imaging speed can be significantly improved through the compressed sensing theory. The complete random subsampling of K-space creates an extremely high degree of incoherent artifacts for simplifying the mathematical calculations. Random sampling of K-space points is generally impractical in all dimensions, because the K-space paths will be smooth only when hardware and physiological considerations have been met. Our goal is to design practical decoherence subsampling models simulating the interference properties of the pure random subsampling until it is possible to quickly gather information. This paper introduces 3 subsampling techniques for K-space data, providing the best efficiency in the production of sparse incoherent artifacts based on the compressed sensing theory. All the proposed methods were simulated on real-life data compared against the MRI results.*

**Keywords:** compressed sampling, X-and Y-axis, contrast, stochastic processes, K-space, sparse representation

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## 1. Introduction

Magnetic Resonance Imaging (MRI) is a noninvasive imaging technique capable of delivering images with various contrast levels from soft tissues and excellent visualization of anatomical structures and physiological functions. However, this imaging method is excessively time-consuming, and a lengthy scan may lead to a severe decline in image quality due to

voluntary and involuntary movements of the patient and other artifacts. The scan time can no longer be reduced due to technological limitations and physiological. The only strategy is to increase the imaging speed is lower data acquisition. In fact, a desirable quality of images can be obtained when the number of data acquired is minimized. Therefore, several imaging techniques have been proposed on how to correctly sample and reconstruct an image through lower

number of data, such as parallel imaging and partial Fourier imaging. Parallel imaging involves multiple coils each with a particular sensitivity to certain parts of the image. The image is reconstructed using the receiver coil sensitivity models and acquired data. In this way, the imaging speed can be increased to about two times. One of the most important elements of parallel imaging is the receiver coil with suitable arrays selected based on their functionality. The coil arrays entail two to eight elements.

The geometric arrangement to achieve the available signal-to-noise ratio (SNR) is extremely difficult. Moreover, it is crucial to maintain the coils distance sensitivity during imaging. Besides, it is critical to determine the specific sensitivity to ensure the image reconstruction in parallel imaging. And that is fulfilled by making additional reference measurements at the beginning of the experiment.

A special reference can be applied periodically by capturing any image. Since the MRI images can entail sparse representation through a linear transform such as wavelet etc. the compressed sensing theory argues that the image can be desirable reconstructed by selecting a small number of k-space data [13,11]. This theory can be used to reduce the duration of MRI scan, which is important in medical imaging.

### **Magnetic resonance imaging (MRI)**

Magnetic resonance imaging is a time-consuming medical procedure. The MRI scanners require large amounts of measurements to construct an image, making the scan too lengthy. Magnetic resonance

imaging does not expose the patients to ionizing radio waves, even though the longer scanning time is not desirable and imposes discomfort to many patients, especially children. The sampling speed is limited to physical (such as the gradient range) and physiological characteristics. Therefore, many researchers have attempted to curtail the amount of incoming data without compromising the image quality. Many of these efforts have taken inspiration from the idea that IMR data are iterative or can be converted repeatedly. Efforts to reduce data acquisition can be called compressed sampling [17].

### **Compressed Sensing (CS)**

Compressed sensing is a new method of sampling, allowing to sample below the Nyquist rate without reducing the reconstruction quality, providing minimum quality loss. This method involves the sparse feature of the signals in various fields. Reduced processing time, cost and hardware requirements are the characteristic features of this method. There are multiple potential applications for this theory in signal processing, including analog to digital conversion, radar, sensor networks and medical imaging. Although compressed sampling is a new method, there have been various strategies proposed so far to reconstruct the original signal. Moreover, there have been numerous criteria for assessing algorithms in signal reconstruction such as speed convergence, reconstruction error, the number of samples required, computational complexity and so on. This algorithm requires to meet three conditions

including sparseness, decoherence and non-linear reconstruction [9,3].

### **Compressed Sensing in MRI (CS-MRI)**

According to advanced mathematical theories in the CS, images with sparse representation can be retrieved by random sampling of k-space data. Random sampling can leave noisy effects. In a sparse space, the main factors stand against the interference of such effects. A non-linear thresholding can recover sparse coefficients and recovered the image. The incoherent sampling procedure has been analyzed and developed through the effects of their artifacts. Incompatibilities are introduced by the random variable density sampling of the coding phase. Reconstruction takes place by minimizing, transformed soft image [13].

Sparseness refers to a sparse transform, i.e. the basic concept for random recovery for sparse representation in a specific area of mathematical transform. To begin with, if the transform area is the image itself, then sparseness refers to relative reduction of the original pixels with a non-zero value. Sparseness is a fundamental limit, generalized from the concept of limited object backup. It is understandable why the restriction on the image space facilitates a sparser sampling in K-space. Sparseness limitations are general because all non-zero coefficients are not collected within a range. Sparseness is more general transform because it only requires to be revealed in an area. Sparseness limitation enables sparser sampling in the K-space [4,5].

## **2. Background research**

### **2.1 Full acquisition of K-space data**

Coils choose a cutting and a displacement in production of a cutting axis and phase displacement produces a cutting along the other axis. This system can specify the status of coils around the direction of signal rotation (phase) by measuring the number of times the magnetic torque cuts the receiver coil (frequency). After collecting the data for each signal status, the information is stored as data points in the onboard computer system. These data points are stored within the K-space. K-space is where the information is stored concerning the signal frequency and location [11,15]. The K-space lines are usually numbered from the lowest number close to the central axis stretching to the highest number on the outer edge. The gradient range of frequency is indicated by field of view (FOV), where the points are usually filled from left to right in sampling. The number of collected points will form the image frequency matrix. An image can be produced from spatial point through the Fourier transform, obtaining the final image. The K-space data are symmetric, i.e. data from the upper half are identical to those from the lower half [17]. The duration of filling K-space is specified by the scan duration, which depends on the iteration time, the matrix phase and the number of stimulations. If the entire K-space data are obtained, there will be an image with ideal contrast and resolution. If part of the K-space data are obtained, however, there will be a low-contrast or low-resolution image. Magnetic resonance imaging requires

longer scan duration to achieve the desired images, because obtaining good quality images to be applied in medical procedures need a greater number of data rows of K-space, which is excessively time-consuming [11]. It can be argued that the main disadvantage of this imaging method is slow speed. And most recently, certain techniques have been adopted to solve this problem to some extent [13].

### **2-2 Partial Fourier**

This method was first proposed by MacFall JR et al. Since the magnetic resonance imaging data are collected in this area, the first and third spatial quarters can be obtained to achieve the rest of data through the data symmetry feature within the Fourier range [14]. The imaging can be accelerating by obtaining half of the K-space data and then reconstruct the other hand of data using the data symmetry in Fourier range, which is known as partial Fourier. In this technique, the greatest reduction factor is equal to 2, and no image can be reconstructed by less than half of the MRI image data. In addition, this method is not resistant to external noise and unwanted elements [12].

### **2.3 Data acquisition with multiple coils**

Known as parallel imaging, this technique was first proposed by Sodickson [16]. In this method, multiple coils are used to simultaneously obtain MRI data, thus enhancing the imaging speed in proportion to the number of coils involved. In addition, each of the coils has a sensitivity function applied to reconstruct the final image [11]. The main disadvantages of this method can

be excessive hardware requirements and reduction factor limited to the number of coils. These techniques were integrated through a set of coils placed side by side and simultaneous acquisition of multiple datasets [6,10]. Accordingly, an array of receiver coils is used to further reduce the data collection time. This method generally differs from other conventional methods of accelerating imaging. The sensitivity of each coil is a function of spatial position. In parallel imaging, data of each section is collected through all receiver coils. If the subset from K-space obtained by each coil is used alone to create the image, it will entail bad representation due to non-compliance with the Nyquist frequency requirement. This problem may be fixed in the image space or K-space. There are various algorithms adopted to reconstruct images in parallel imaging such as sensitivity coding (SENSE), parallel imaging spatial sensitivity (SMASH), (PILS) and (GRAPPA) [2,10]. Some of the final image reconstruction algorithms take place within the Fourier range and in others such as GRAPPA. One of the most important components of parallel imaging is the receiver coils with appropriate arrays, which are selected according to the application. Coils arrays include, for example, 2 to 8 elements [10].

### **2.4 Adaptive subsampling**

This technique involves adaptive algorithms such as singular value decomposition (SVD) and principal component analysis (PCA) for sparse representation of MRI images. These methods were described fully in references [8,18]. The main advantage of this method is

the reduction of the errors in the image reconstruction. Moreover, these adaptive algorithms can be adopted in most MRI applications such as encephalography, cardiography and angiography. The main disadvantage of these methods is the longer processing time. Given that very large volumes of data are involved in practical tasks, we need much faster algorithms [8].

### 3. Methodology

This paper intended to propose several strategies to accelerate the acquisition of MRI images through the compressed sensing theory. For this purpose, there has been some research conducted so far to improve the three conditions required by the compressed sensing theory for reconstruction of MRI images. We used a MATLAB code [13] upon which the proposed algorithms were applied. In the same procedure, several subsampling models were considered for sampling of K-space data in the compressed sensing theory, where all these models met the decoherence conditions. Finding the best sampling is an essential step to be taken in the compressed sensing theory. When subsampling the K-space, the image contrast or the K-space center and incoherent artifacts produced by random sampling should be considered.

### 4. The proposed method

Due to the rapid growth of compressed sensing theory and adoption of new algorithms to improve this theory, it was crucial to carry on the previous studies so as to implement and develop new algorithms. In this article, effort was made to review the

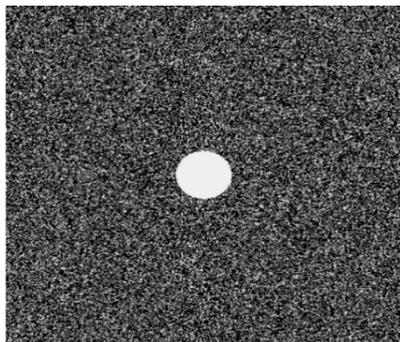
relevant literature and select SparseMRI as the main reference [13]. Moreover, all the proposed methods served to improve this method or apply new algorithms based on it.

#### 4.1 K-space sampling models in compressed sensing

Finding the best sampling is an essential step to be taken in the compressed sensing theory. When subsampling the K-space, the image contrast or the K-space center and incoherent artifacts produced by random sampling should be considered. This article introduced subsampling models for K-space data with the best efficiency in the production of sparse incoherent artifacts applied within the compressed sensing theory. Evidently, equidistant sampling would not satisfy the requirements in compressed subsampling. In subsampling, it is crucial to examine the decoherence conditions in the specific sparse range as well as the main lobe to side lobe of impulse response (PSF) [1]. Sampling with random functions such as Gaussian, Bernoulli and partial Fourier will fulfill the requirements in the compressed sensing theory, even though the main disadvantage of these methods are difficult implementation on hardware. For sampling in MRI images, it should be noted that only the phase coding directions (PC) are effective at the time of scanning. Hence, we can fully obtain the readout lines. The methods discussed in this for K-space data sampling were as follows:

The full sampling of K-space center and application of Gaussian probability function for the rest of this space. In this method, the decoherence conditions and image contrast

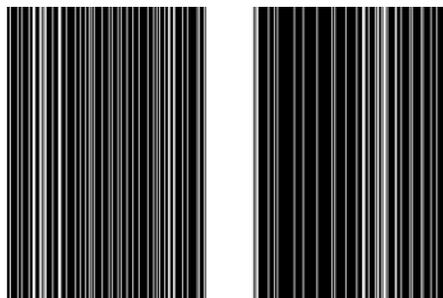
were met at the early stages of the algorithm. Hence, this method guarantees the conditions of compressed sensing theory. The figure below demonstrates 33% of total K-space data using the sampling model. This was employed as a basis for comparing the newly proposed methods.



**Fig.1.** The subsampling of K-space data using the Gaussian probability function met the requirements of the compressed sensing theory

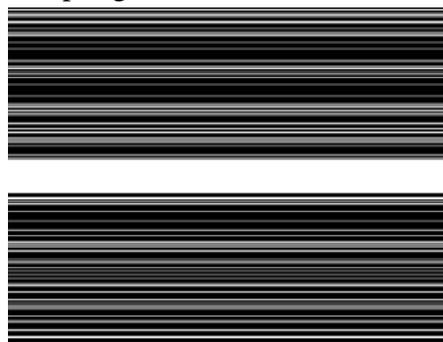
- 1- The previous method proposed for the subsampling of 3D K-space is effective because subsampling was carried out in two directions and the coding phase in the three-dimensional MRI data is in two dimensions. It is better to subsample two-dimensional data in one single direction while the other direction (the direction of frequency coding) is fully obtained. In this method, subsampling is done in the direction of the X axis randomly or through the Gaussian probability function while sampling is completely done in the other direction. This method has very low computational complexity and its implementation on hardware is simpler. All conditions of compressed sampling are not fulfilled in this sampling model, although this model yields better results

in the first iterations for Zero Filling (ZF). The figure below displays the sampling model.



**Fig.2.** Sub-sampling model of K-space data using the random function in one direction

- 2- Changing axes in the second method implies that sampling was fully in direction of X-axis and randomly in the direction of the Gaussian probability function for the y-axis. In this sampling procedure, the directions of phase coding and readout were switched and the results were the same as expected from the previous method. The following figure displays the sub-sampling model used in this method.

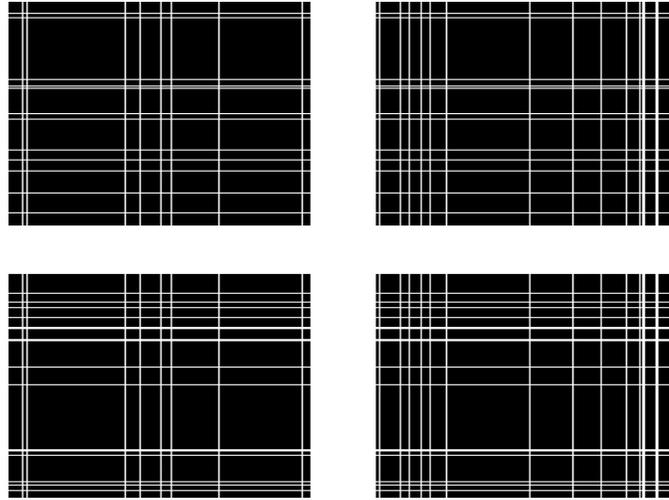


**Fig.3.** Sub-sampling model of K-space data, using the random function in the direction of y-axis

- 3- This method involved the total half of K-space data from the second and third methods. In this way, effort was made to

obtain mostly the data of K-space center. Hence, the reconstructed image in this situation has a good contrast between the early iterations, indicating the desirability of this model for the ZF method. However, RIP conditions are completely

met due to lack of random structure in this model, rendering a reconstructed image with misrepresented artifacts. The following figure displays this model with acceleration factor of 3 (3-fold).



**Fig.4.** Sub-sampling model of K-space data, ideal for the ZF method

#### 4.2 Non-linear conjugate gradient (NLCG)

This algorithm was adopted from the article of Sparse MRI to solve the non-constrained equation in the final image reconstruction using the compressed sensing theory [13].

$$\arg \min_m \|F_u m - y\|_2^2 + \lambda \|\psi(m)\|_1$$

In the above equation:  $m$ : Reconstructed image  $y$ : K-space data measured in the scanner

$\lambda$ : Setting parameter  $\psi$ : Linear function that converts pixel representation to sparse representation

Where  $\lambda$  is the setting parameter applied to maintain compromise between sparseness and error. The above equation is solved through the NLCG algorithm, and the above equation is known as the cost function [7].

#### 5. Analysis of Results

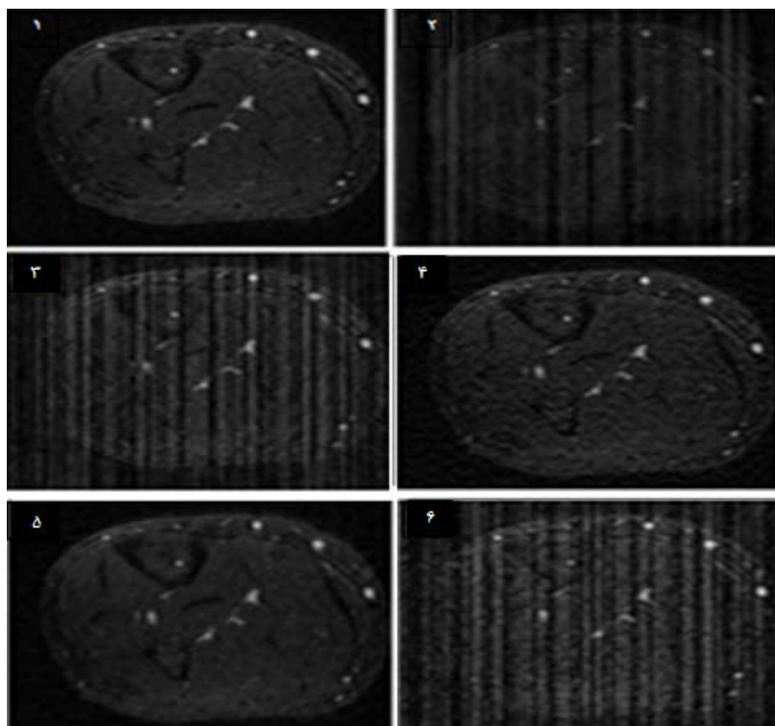
Due to the rapid growth of compressed sensing theory and adoption of new algorithms to improve this theory, it was crucial to carry on the previous studies so as to implement and develop new algorithms. That is why the proposed methods were compared against Sparse MRI and zero filling (ZF). All the proposed methods were simulated on real-life data compared against the MRI results. In all simulations, 0.3 of K-

space data was used. To compare the proposed methods together with previous methods through several criteria such as the similarity of the original image, signal to noise ratio (SNR) and PSNR. Moreover, advantages and disadvantages of each set of data for the proposed methods and previously proposed methods were discussed.

### 5.1. First data

The methods described in the previous section were applied on magnetic resonance imaging data. The results obtained from complete sampling methods were examined through the Gaussian random process and zero filling (ZF). Given that these images lie

within the pixels entailing sparse representation, this area was used for sparseness. Furthermore, Table 1 displays the numerical results, where proposed methods 2 and 3 outperformed the other algorithms [13]. Figure 5 displays the reconstructed images with the original images. The images involved 0.3 of K-space data. Numerical results have been given in Table 1. Computational time for reconstruction of images through NLCG algorithm was about 40 seconds due to the expanded volume of images. ZF method does not require special reconstruction time. The non-acquired data in this method were zero filled and the run-time was about 5 seconds.



**Fig.5.** With the order from left to right, top, original image with the total K-space data, images reconstructed through subsampling with Gaussian in two directions, the proposed methods 1, 2 and 3 image reconstructions with ZF.

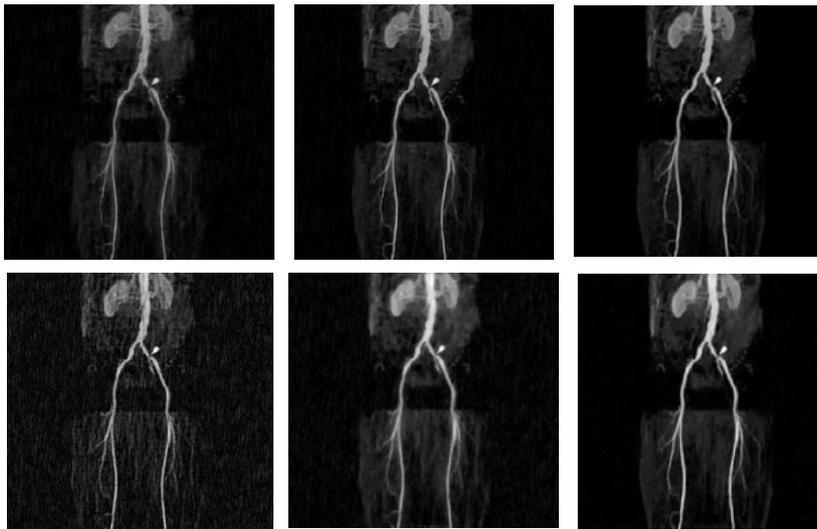
**Table.1.** The numerical results obtained from application of the proposed algorithm and the previous methods on the second dataset

Methods:	SNR	PSNR	SSIM	Time(S)
<b>Proposed method 1</b>	21.17	23.17	84%	40
<b>Proposed method 2</b>	18.46	26.49	91%	40
<b>Proposed method 3</b>	18.49	26.92	93%	40
<b>Gaussian Sampling Method</b>	17.92	24.02	78%	42
<b>Zero Filling (ZF)</b>	11.32	17.79	61%	5

### 5-2 Second data

To obtain better results, the proposed algorithms were implemented on this series of data. The results showed that the proposed methods 1 and 3 are better than others [18]. Figure 6 shows the images reconstructed by algorithms together with the numerical results. Given that the images are the sparse pixels, there is more consistency with the

compressed sensing theory. The results also showed such consistency. Computational time for reconstruction of images through NLCG algorithm was about 20 seconds due to the expanded volume of images. ZF method does not require special reconstruction time. The non-acquired data in this method were zero filled and the run-time was about 2 seconds.



**Fig.6.** Order from left to right, top, original image with the total K-space data, images reconstructed through methods 1, 2, 3 and 4 subsampling through Gaussian in two directions, image obtained by ZF.

**Table.2.** The numerical results obtained from application of the proposed algorithm and the previous methods on the third dataset

Methods:	SNR	PSNR	SSIM	Time(S)
<b>Proposed method 1</b>	19.88	27.92	94%	20
<b>Proposed method 2</b>	18.06	25.98	83%	20
<b>Proposed method 3</b>	18.86	26.34	89%	20
<b>Gaussian sampling method</b>	17.82	24.26	78%	21
<b>Zero filling (ZF)</b>	12.52	21.75	66%	2

## 6. Conclusion

Given that many aspects and applications of compressed sampling have not yet fully known, this paper intended to examine the application of this theory in the magnetic resonance imaging through a review of relevant literature. The essential conditions for compressed sensing theory were first evaluated and then effort was made to assess the entire features and requirements of the theory. In general, this theory can be applied on MRI images given the fact that most MRI images have good sparse areas, and sparseness is the basic condition required by compressed sensing theory. The second condition is the subsampling model of K-space data.

This paper proposed three methods, given that a great deal of work has been done in this realm and both the magnetic resonance imaging hardware and decoherence in compressed sampling theory should have been fulfilled. Each of the proposed methods were fully investigated and compared against the previous methods. The results showed that if the subsampling models such as Gaussian random processes was conducted in the coding phase direction and

frequency coding direction, the quality of reconstructed images will be desirable. In addition, given that the frequency coding was not effective in the data acquisition time, this sampling method could curtail the volume and data acquisition speed as well. Many non-linear reconstruction algorithms have been proposed for compressed sampling theory in the field of mathematics, where this algorithm should have great convergence and high implementation speed. This paper involved the nonlinear conjugate gradient algorithm (NLCG) after reviewing several reconstruction algorithms.

The sparse areas vary with respect to the characteristics of images. For example, brain images have sparse representation in wavelet domain, while the heart images within the cosine transform entail a better sparse representation. Finally, great effort was made to focus more on sampling models of K-space data. In this regard, taking into account the limitations of hardware devices, magnetic resonance imaging conditions were proposed for decoherence compressed sampling methods. The results indicated that the newly proposed method improves the quality of reconstructed images better with identical data.

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